

SLAVNOV-TAYLOR IDENTITY FOR NONEQUILIBRIUM QUARK-GLUON PLASMA

K. OKANO*

Department of Physics, Osaka City University, Sumiyoshi-ku, Osaka 558-8585, Japan

Abstract

Within the closed-time-path formalism of nonequilibrium QCD, we derive a Slavnov-Taylor (ST) identity for the gluon polarization tensor. The ST identity takes the same form both in Coulomb and covariant gauges. Application to quasi-uniform quark-gluon plasma (QGP) near equilibrium or nonequilibrium quasistationary QGP is made.

1 Introduction

Much interest is devoted to the physics of a deconfinement phase of hadronic matter (quark-gluon plasma, QGP), with both strong experimental and theoretical research going on. A theoretical understanding of this new phase of matter can be gained in the framework of hot QCD supplemented with a perturbative approach and important progress has been made during the last decade [1]. At early stages, the QGP is treated as a thermally and/or chemically equilibrium system. Studies of the QGP as a nonequilibrium system have recently begun.

The symmetry transformation which leaves the Lagrangian invariant plays an important role. Each continuous symmetry leads to relations between Green functions. As an example, we take up the BRST invariance of the QCD Lagrangian, which leads to Slavnov-Taylor (ST) identities. In a previous work [2], we derived a ST identity for gluon polarization tensor in equilibrium QGP within the imaginary-time formalism [3,4]. As an application of it, we dealt with damping rates for soft gluons. The purpose of this letter is to derive a ST identity for the case of nonequilibrium QGP. We employ the closed-time-path (CTP) formalism [5,6,7] of nonequilibrium QCD. It turns out that the deduced ST identity takes the same form both in Coulomb and covariant gauges.

2 Preliminaries

We start with the QCD Lagrangian density in Coulomb gauge:

$$\begin{aligned} \mathcal{L}(A, \omega, \bar{\omega}) = & -\frac{1}{4}F_{\mu\nu}^a(x)F^{a,\mu\nu}(x) - \frac{\lambda}{2}(\nabla \cdot \mathbf{A}^a(x))(\nabla \cdot \mathbf{A}^a(x)) \\ & - \partial^\mu \bar{\omega}^a(x) D_\mu^{ac}(A(x)) \omega^c(x), \end{aligned} \quad (1)$$

*E-mail: okano@sci.osaka-cu.ac.jp

$$F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + gf^{abc}A_\mu^b(x)A_\nu^c(x), \quad (2)$$

where a, b, c are the color indices, $D_\mu^{ac}(A(x)) \equiv \delta^{ac}\partial/\partial x^\mu + gf^{abc}A_\mu^b(x)$, and $x^\mu \equiv x^\mu - x_0 n^\mu$ with $n^\mu = (1, \mathbf{0})$. ω^a and $\bar{\omega}^a$ are the FP-ghost fields (in Coulomb gauge). The quark sector does not play any role for our purpose.

The CTP formalism is constructed by introducing an oriented closed-time path C ($= C_1 \oplus C_2$) in a complex-time plane, that goes from $-\infty$ to $+\infty$ (C_1) and then returns from $+\infty$ to $-\infty$ (C_2). The time arguments of the fields are on the time path C . A field with time argument on C_1 [C_2] is called a type-1 [type-2] field. A classical contour action is written in the form

$$\begin{aligned} & \int_C dx_0 \int d^3x \mathcal{L}(A, \omega, \bar{\omega}) \\ &= \int_{-\infty}^{+\infty} dx_0 \int d^3x [\mathcal{L}(A_1, \omega_1, \bar{\omega}_1) - \mathcal{L}(A_2, \omega_2, \bar{\omega}_2)] \\ &\equiv \int_{-\infty}^{+\infty} dx_0 \int d^3x \hat{\mathcal{L}}(x), \end{aligned} \quad (3)$$

where the subscripts “1” and “2” stand for the type of fields and $\hat{\mathcal{L}}$ is sometimes called a hat-Lagrangian [8].

The (full) gluon propagator $G_{rs}^{ab,\mu\nu}(x, y)$ is defined by the statistical average of the time-path-ordered product T_C of gluon fields:

$$G_{rs}^{ab,\mu\nu}(x, y) \equiv \text{Tr} [T_C(A_r^{a,\mu}(x)A_s^{b,\nu}(y))\rho] \equiv \langle T_C(A_r^{a,\mu}(x)A_s^{b,\nu}(y)) \rangle. \quad (4)$$

Here ρ is the density matrix [7]:

$$\rho = \int \mathcal{D}A_\mu^a(\mathbf{x}) \mathcal{D}A_\nu^b(\mathbf{x}) |A_\mu^a(\mathbf{x})\rangle \rho(A_\mu^a, A_\nu^b) \langle A_\nu^b(\mathbf{x})|, \quad (5)$$

where $|A_\mu^a(\mathbf{x})\rangle$ is the eigenstate of the in-field operator $A_\mu^a(x_0 = -\infty, \mathbf{x})$.¹ The (full) ghost propagator $\tilde{G}_{rs}^{ab}(x, y)$ reads

$$\tilde{G}_{rs}^{ab}(x, y) \equiv \langle T_C(\omega_r^a(x)\bar{\omega}_s^b(y)) \rangle. \quad (6)$$

The bare ghost propagator $\tilde{\Delta}_{rs}^{ab}(x, y)$ is constructed from Eq.(3) with Eq.(1):

$$\tilde{\Delta}_{rs}^{ab}(x, y) = \delta^{ab}(-1)^{r-1}\delta_{rs} \int \frac{d^4P}{(2\pi)^4} \frac{i}{p^2} e^{-iP \cdot (x-y)} \quad (7)$$

with no summation over r , $P^\mu = (p_0, \mathbf{p})$, and $p \equiv |\mathbf{p}|$. The gluon-ghost vertex factor for the “type-1” and “type-2” vertices can be read off from $\hat{\mathcal{L}} \ni (-1)^t gf^{abc}\partial^\mu \omega_t^a A_{t,\underline{\mu}}^b \omega_t^c$:

$$\mathcal{V}_t = (-1)^{t-1} gf^{abc} P^\mu \quad (t = 1, 2), \quad (8)$$

where $P^\mu = P^\mu - p_0 n^\mu (= (0, \mathbf{p}))$ with \mathbf{p} the momentum of the out-going FP-ghost, and ‘ $\underline{\mu}$ ’ is a suffix of $A_{t,\underline{\mu}}^b$.

¹In the case of covariant gauge, the FP-ghost fields should also be included.

We introduce here the generating functional:

$$\begin{aligned}\hat{Z}[J, \bar{\xi}, \xi] &= \int \prod_{r=1}^2 \mathcal{D}A_{r,\mu}^a \mathcal{D}\omega_r^a \mathcal{D}\bar{\omega}_r^a \\ &\times \exp \left[i \int_{-\infty}^{+\infty} d^4z \left\{ \hat{\mathcal{L}} + \sum_{t=1}^2 (J_t^{a,\mu} A_{t,\mu}^a + \bar{\xi}_t^a \omega_t^a \right. \right. \\ &\left. \left. + \bar{\omega}_t^a \xi_t^a) \right\} \right] \rho(A_\mu^a, A_\nu^b),\end{aligned}\quad (9)$$

where $J, \bar{\xi}, \xi$ are (classical) source functions. Equation (9) is to be computed with periodic boundary conditions, $A_{1,\mu}^a(x_0 = -\infty, \mathbf{x}) = A_{2,\mu}^a(x_0 = -\infty, \mathbf{x})$, $\omega_1^a(x_0 = -\infty, \mathbf{x}) = \omega_2^a(x_0 = -\infty, \mathbf{x})$ and $\bar{\omega}_1^a(x_0 = -\infty, \mathbf{x}) = \bar{\omega}_2^a(x_0 = -\infty, \mathbf{x})$. These conditions come from the trace operation [4,6]. Note that, inspite of the fact that the ghost fields are fermionic, they obey a periodic boundary condition [4,9]. As is stated above after Eq.(5), A_μ^a and A_ν^b of ρ in Eq.(9) are the eigenvalues of the in-fields, i.e., the fields at $x_0 = -\infty$, so that $A_\mu^a = A_{1,\mu}^a = A_{2,\mu}^a$ etc. $\hat{Z}[J, \bar{\xi}, \xi]$ generates the above defined (full) propagators through

$$G_{rs}^{ab,\mu\nu}(x, y) = \left. \frac{\delta \ln \hat{Z}[J, \bar{\xi}, \xi]}{i\delta J_{s,\nu}^b(y) i\delta J_{r,\mu}^a(x)} \right|_{J=\bar{\xi}=\xi=0}, \quad (10)$$

$$\tilde{G}_{rs}^{ab}(x, y) = \left. \frac{\delta \ln \hat{Z}[J, \bar{\xi}, \xi]}{i\delta \xi_s^b(y) i\delta \bar{\xi}_r^a(x)} \right|_{J=\bar{\xi}=\xi=0}. \quad (11)$$

The Lagrangian density \mathcal{L} is invariant [10] under the BRST transformation:

$$\begin{aligned}\delta A_\mu^a &= \zeta D_\mu^{ac}(A) \omega^c, \quad \delta \omega^a = -\frac{1}{2} g \zeta f^{abc} \omega^b \omega^c, \\ \delta \bar{\omega}^a &= \lambda \zeta \nabla \cdot \mathbf{A}^a,\end{aligned}\quad (12)$$

where ζ is a Grassmann-number parameter. Throughout in the sequel, we deal with the systems whose density matrix ρ is invariant under the BRST transformation. Using these facts for Eq.(9), we obtain

$$\begin{aligned}\int d^4z \mathcal{B}(z) \hat{Z}[J, \bar{\xi}, \xi] &= 0, \\ \mathcal{B}(z) &= \sum_{t=1}^2 \left[J_t^{a,\mu}(z) D_\mu^{ac} \left(\frac{\delta}{i\delta J_t(z)} \right) \frac{\delta}{i\delta \bar{\xi}_t^c(z)} + \lambda \xi_t^a(z) \frac{\partial}{\partial z_\mu} \frac{\delta}{i\delta J_t^{a,\mu}(z)} \right. \\ &\quad \left. + \bar{\xi}_t^a(z) \frac{g}{2} f^{abc} \frac{\delta}{i\delta \bar{\xi}_t^b(z)} \frac{\delta}{i\delta \bar{\xi}_t^c(z)} \right].\end{aligned}\quad (13)$$

3 Slavnov-Taylor identity

In the following we deal with systems, for which $\langle A_r^{a,\mu}(x) \rangle = 0$ holds. Computing

$$\frac{\delta}{i\delta J_{r,\mu}^a(x)} \frac{\delta}{i\delta \xi_s^b(y)} \int d^4z \mathcal{B}(z) \ln \hat{Z}[J, \bar{\xi}, \xi] \Big|_{J=\bar{\xi}=\xi=0}, \quad (14)$$

by using Eq.(13), we obtain

$$\begin{aligned} & \frac{\partial}{\partial x_\mu} \tilde{G}_{rs}^{ab}(x, y) + g f^{acd} \langle T_C(A_r^{c,\mu}(x) \omega_r^d(x) \bar{\omega}_s^b(y)) \rangle \\ & + \lambda \frac{\partial}{\partial y^\mu} G_{rs}^{ab,\mu\nu}(x, y) = 0. \end{aligned} \quad (15)$$

Here the gluon-ghost three-point function $\langle T_C(A_r^{c,\mu}(x) \omega_r^d(x) \bar{\omega}_s^b(y)) \rangle$ is

$$\langle T_C(A_r^{c,\mu}(x) \omega_r^d(x) \bar{\omega}_s^b(y)) \rangle = \frac{\delta \ln \hat{Z}[J, \bar{\xi}, \xi]}{i \delta \xi_s^b(y) i \delta J_{r,\mu}^c(x) i \delta \bar{\xi}_r^d(x)} \Big|_{J=\bar{\xi}=\xi=0} \quad (16)$$

with no summation over r . Note that the third term on the right-hand side of Eq.(13) does not contribute to Eq.(15). The gluon-ghost three-point function (16) may be written as

$$\begin{aligned} & g f^{acd} \langle T_C(A_r^{c,\mu}(x) \omega_r^d(x) \bar{\omega}_s^b(y)) \rangle \\ & = i(-1)^r \int d^4 w \tilde{\Pi}_{rt}^{ac,\mu}(x, w) \tilde{G}_{ts}^{cb}(w, y). \end{aligned} \quad (17)$$

$\tilde{\Pi}_{rt}^{ac,\mu}(x, w)$ here is related to a ghost self-energy part $\tilde{\Pi}_{rt}^{ac}(x, w)$ through

$$i \frac{\partial}{\partial x^\mu} \tilde{\Pi}_{rt}^{ac,\mu}(x, w) = \tilde{\Pi}_{rt}^{ac}(x, w). \quad (18)$$

From now on we use a bold-face letter to denote a (8×8) matrix in color space, while a caret ‘^’ to denote a (2×2) matrix in “type” space. For example, a full gluon propagator $\hat{\mathbf{G}}^{\mu\nu}(x, y)$ is a (8×8) matrix in color space with matrix element $\hat{G}^{ab,\mu\nu}(x, y)$, which is a (2×2) matrix in “type” space with matrix element $G_{rs}^{ab,\mu\nu}(x, y)$. Then by using Eq.(17), Eq.(15) can be written as

$$\frac{\partial}{\partial x_\mu} \hat{\mathbf{G}}(x, y) - i \hat{\tau} \int d^4 w \hat{\mathbf{\Pi}}^{\mu}(x, w) \hat{\mathbf{G}}(w, y) + \lambda \frac{\partial}{\partial y^\mu} \hat{\mathbf{G}}^{\mu\nu}(x, y) = 0, \quad (19)$$

where $\hat{\tau} = \text{diag}(1, -1)$.

We multiply Eq.(19) by the inverse full gluon propagator $\hat{\mathbf{G}}_{\nu\mu}^{-1}(v, x)$ from the left, by the inverse full ghost propagator $\hat{\mathbf{G}}^{-1}(y, z)$ from the right, and then integrate over x and y , to obtain

$$-\frac{\partial}{\partial z_\mu} \hat{\mathbf{G}}_{\nu\mu}^{-1}(v, z) - i \int d^4 x \hat{\mathbf{G}}_{\nu\mu}^{-1}(v, x) \hat{\tau} \hat{\mathbf{\Pi}}^{\mu}(x, z) - \lambda \frac{\partial}{\partial v^\mu} \hat{\mathbf{G}}^{-1}(v, z) = 0. \quad (20)$$

Here we recall the Schwinger-Dyson equations:

$$\begin{aligned} \hat{\mathbf{G}}^{\mu\nu}(x, y) &= \hat{\mathbf{\Delta}}^{\mu\nu}(x, y) - i \int d^4 z \int d^4 w \hat{\mathbf{\Delta}}^{\mu\rho}(x, z) \hat{\mathbf{\Pi}}_{\rho\sigma}(z, w) \hat{\mathbf{G}}^{\sigma\nu}(w, y) \\ &= \hat{\mathbf{\Delta}}^{\mu\nu}(x, y) - i \int d^4 z \int d^4 w \hat{\mathbf{G}}^{\mu\rho}(x, z) \hat{\mathbf{\Pi}}_{\rho\sigma}(z, w) \hat{\mathbf{\Delta}}^{\sigma\nu}(w, y), \end{aligned} \quad (21)$$

$$\begin{aligned} \hat{\mathbf{G}}(x, y) &= \hat{\mathbf{\Delta}}(x, y) - i \int d^4 z \int d^4 w \hat{\mathbf{\Delta}}(x, z) \hat{\mathbf{\Pi}}(z, w) \hat{\mathbf{G}}(w, y) \\ &= \hat{\mathbf{\Delta}}(x, y) - i \int d^4 z \int d^4 w \hat{\mathbf{G}}(x, z) \hat{\mathbf{\Pi}}(z, w) \hat{\mathbf{\Delta}}(w, y), \end{aligned} \quad (22)$$

where $\hat{\Delta}^{\mu\nu}(\hat{\Delta})$ is the bare gluon (ghost) propagator. Then we have

$$\begin{aligned}\hat{G}_{\mu\nu}^{-1}(x, y) &= \hat{\Delta}_{\mu\nu}^{-1}(x, y) + i\hat{\Pi}_{\mu\nu}(x, y) \\ &= -i\mathbf{I}\hat{\tau}\left(g_{\mu\nu}\partial_x^2 - \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} + \lambda\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}\right)\delta^4(x - y) + i\hat{\Pi}_{\mu\nu}(x, y)\end{aligned}\quad (23)$$

$$\begin{aligned}\hat{G}^{-1}(x, y) &= \hat{\Delta}^{-1}(x, y) + i\hat{\Pi}(x, y) \\ &= i\mathbf{I}\hat{\tau}\nabla_x^2\delta^4(x - y) + i\hat{\Pi}(x, y),\end{aligned}\quad (24)$$

where \mathbf{I} is a (8×8) unit matrix in color space. Substituting Eqs.(23) and (24) into Eq.(20), we finally obtain

$$\begin{aligned}&-i\frac{\partial}{\partial z_\mu}\hat{\Pi}_{\nu\mu}(v, z) - \left(g_{\nu\mu}\partial_v^2 - \frac{\partial}{\partial v^\nu}\frac{\partial}{\partial v^\mu}\right)\hat{\Pi}^\mu(v, z) \\ &+ \int d^4x \hat{\Pi}_{\nu\mu}(v, x) \hat{\tau} \hat{\Pi}^\mu(x, z) = 0.\end{aligned}\quad (25)$$

This is a desired Slavnov-Taylor identity for gluon polarization tensor in Coulomb gauge. Note that, in the course of derivation, the terms which explicitly depend on the Coulomb gauge parameter λ (Eqs.(20) and (23)) cancel out with the help of Eq.(18).

Through similar procedure to the above derivation, one can derive a covariant-gauge counterpart of Eq.(25). As a matter of fact, it takes the same form as Eq.(25).

4 Out-of-equilibrium QGP

Here we deal with quasi-uniform QGP near equilibrium or nonequilibrium quasistationary QGP, which we simply refer to as out-of-equilibrium QGP. Out-of-equilibrium QGP is characterized by two different spacetime scales: microscopic or quantum-field-theoretical and macroscopic or statistical. The first scale, the microscopic-correlation scale, characterizes the range of radiative corrections to reactions taking places in the QGP, while the second scale measures the relaxation of the QGP. A well-known intuitive picture for dealing with such system is to divide spacetime into many “cells” whose characteristic size, L^μ ($\mu = 0, \dots, 3$), is in between microscopic and macroscopic scales. It is assumed that the correlation between different cells is negligible in the sense that microscopic or elementary reactions can be regarded as taking place in a single cell. On the other hand, in a single cell, relaxation phenomena are negligible.

The above intuitive picture may be implemented as follows. Let $\Delta(v, z)$ be a generic propagator. For an out-of-equilibrium QGP, $\Delta(v, z)$, with $v - z$ fixed, does not change appreciably in the region $|X^\mu - X_0^\mu| \lesssim L^\mu$, where $X^\mu \equiv (v^\mu + z^\mu)/2$ is the midpoint and X_0^μ is an arbitrary spacetime point. The self-energy part $\Pi(v, z)$ enjoys a similar property. Thus, X^μ may be used as a label for the spacetime cells and is called the “macroscopic spacetime coordinates.” On the other hand, relative spacetime coordinates $v^\mu - z^\mu$ describe microscopic reactions taking place in a single spacetime cell. A Fourier transformation with

respect to the relative coordinates $v^\mu - z^\mu$ yields

$$\Delta(X; P) \equiv \int d^4(v - z) e^{iP \cdot (v - z)} \Delta(v, z), \quad (26)$$

together with a similar formula for Π . The above observation shows that P^μ ($\gtrsim 1/L^\mu$) in Eq.(26) can be regarded as the momentum of the quasiparticle participating in the microscopic reaction under consideration. Thus $\Delta(X; P)$ and $\Pi(X; P)$ vary slowly in X . Then, we employ the derivative expansion,

$$\begin{aligned} \Delta(X; P) = & \left[1 + (X - Y)^\sigma \frac{\partial}{\partial Y^\sigma} \right. \\ & \left. + \frac{1}{2!} (X - Y)^\rho (X - Y)^\sigma \frac{\partial}{\partial Y^\rho} \frac{\partial}{\partial Y^\sigma} + \dots \right] \Delta(Y; P), \end{aligned} \quad (27)$$

together with a similar formula for $\Pi(X; P)$.

Fourier transforming Eq.(25) on $v - z$ and carrying out the derivative expansion, we obtain

$$\begin{aligned} & P^\mu \hat{\Pi}_{\nu\mu}(X; P) + (g_{\nu\mu} P^2 - P_\nu P_\mu) \hat{\Pi}^\mu(X; P) + \hat{\Pi}_{\nu\mu}(X; P) \hat{\tau} \hat{\Pi}^\mu(X; P) \\ & + \frac{1}{2i} \frac{\partial}{\partial X_\mu} \hat{\Pi}_{\nu\mu}(X; P) + \frac{1}{2i} \left\{ (g_{\nu\mu} P^2 - P_\nu P_\mu) \hat{\tau} + \hat{\Pi}_{\nu\mu}, \hat{\tau} \hat{\Pi}^\mu \right\}_{X, P} \\ & - \frac{1}{4} \left(g_{\nu\mu} \partial_X^2 - \frac{\partial}{\partial X^\nu} \frac{\partial}{\partial X^\mu} \right) \hat{\Pi}^\mu(X; P) \\ & + \frac{1}{2!} \left(\frac{1}{2i} \right)^2 \left[\left\{ \frac{\partial}{\partial X_\lambda} \hat{\Pi}_{\nu\mu}, \hat{\tau} \frac{\partial}{\partial P^\lambda} \hat{\Pi}^\mu \right\}_{X, P} - \left\{ \frac{\partial}{\partial P^\lambda} \hat{\Pi}_{\nu\mu}, \hat{\tau} \frac{\partial}{\partial X_\lambda} \hat{\Pi}^\mu \right\}_{X, P} \right] \\ & + O((\partial_X)^3) = 0, \end{aligned} \quad (28)$$

where Poisson bracket $\{\dots, \dots\}_{X, P}$ is defined by

$$\left\{ \hat{\mathbf{A}}(X; P), \hat{\mathbf{B}}(X; P) \right\}_{X, P} \equiv \frac{\partial \hat{\mathbf{A}}(X; P)}{\partial X_\mu} \frac{\partial \hat{\mathbf{B}}(X; P)}{\partial P^\mu} - \frac{\partial \hat{\mathbf{A}}(X; P)}{\partial P^\mu} \frac{\partial \hat{\mathbf{B}}(X; P)}{\partial X_\mu}. \quad (29)$$

In view of perturbation theory, $\hat{\Pi}^\mu$ in Coulomb gauge, Eq.(28), is a diagonal matrix in “type space” since the bare ghost propagator $\hat{\Delta}$ is diagonal.

In the following, we restrict our concern to the leading parts of Eq.(28),

$$P^\mu \hat{\Pi}_{\nu\mu}(X; P) + (g_{\nu\mu} P^2 - P_\nu P_\mu) \hat{\Pi}^\mu(X; P) + \hat{\Pi}_{\nu\mu}(X; P) \hat{\tau} \hat{\Pi}^\mu(X; P) = 0. \quad (30)$$

In the case of Coulomb gauge, $\hat{\Pi}_{\nu\mu}(X; P)$ is usually decomposed as

$$\begin{aligned} \hat{\Pi}_{\nu\mu}(X; P) = & \mathcal{P}_{\nu\mu}^T(\hat{\mathbf{p}}) \hat{\Pi}^T(X; P) + n_\nu n_\mu \hat{\Pi}^L(X; P) \\ & + \frac{p_0}{p} \left(\hat{P}_\nu n_\mu + n_\nu \hat{P}_\mu \right) \hat{\Pi}^C(X; P) - \hat{P}_\nu \hat{P}_\mu \hat{\Pi}^D(X; P), \end{aligned} \quad (31)$$

where $\mathcal{P}_{\nu\mu}^T(\hat{\mathbf{p}})$ is the transverse projection operator:

$$\mathcal{P}_{\nu\mu}^T(\hat{\mathbf{p}}) \equiv g_{\underline{\nu}\underline{\mu}} + \hat{P}_\nu \hat{P}_\mu. \quad (32)$$

Here $g_{\underline{\nu}\underline{\mu}} \equiv -\sum_{i,j=1}^3 g_{\nu i} g_{\mu j} \delta^{ij}$ and $\hat{P}^\mu \equiv (0, \hat{\mathbf{p}})$ with $\hat{\mathbf{p}} \equiv \mathbf{p}/p$.

Substituting Eq.(31) into Eq.(30), we obtain

$$\hat{\Pi}^L = \hat{\Pi}^C + \frac{p}{p_0}(pn_\mu + p_0\hat{P}_\mu)\hat{\Pi}^\mu - \left(\frac{1}{p_0}n_\mu\hat{\Pi}^L + \frac{1}{p}\hat{P}_\mu\hat{\Pi}^C\right)\hat{\tau}\hat{\Pi}^\mu, \quad (33)$$

$$\begin{aligned} \hat{\Pi}^L &= -\frac{p^2}{p_0^2}\hat{\Pi}^D + 2\frac{p}{p_0}(pn_\mu + p_0\hat{P}_\mu)\hat{\Pi}^\mu + \left(-\frac{1}{p_0}n_\mu\hat{\Pi}^L + \frac{p}{p_0^2}\hat{P}_\mu\hat{\Pi}^D\right. \\ &\quad \left.- \frac{1}{p_0}\left(\frac{p_0}{p}\hat{P}_\mu + n_\mu\right)\hat{\Pi}^C\right)\hat{\tau}\hat{\Pi}^\mu. \end{aligned} \quad (34)$$

Note that the identities (33) and (34) are valid to all orders in perturbation theory. Ward identity in QED plays a key role in the formal discussion of the theory and simplifies the practical calculation. ST identity derived here is expected to play equally important role in out-of-equilibrium QCD.

Let $\sum_{n=1}^\infty g^{2n}\hat{\Pi}_{\nu\mu}^{(2n)}$ $[\sum_{n=1}^\infty g^{2n}\hat{\Pi}_\mu^{(2n)}]$ be a perturbation series of $\hat{\Pi}_{\nu\mu}$ $[\hat{\Pi}_\mu]$. Substituting this into Eqs.(33) and (34), we obtain

$$\begin{aligned} \hat{\Pi}^{L(2n)} &= \hat{\Pi}^{C(2n)} + \frac{p}{p_0}(pn_\mu + p_0\hat{P}_\mu)\hat{\Pi}^{(2n)\mu} \\ &\quad - \sum_{m=2}^{2n-2} \left(\frac{1}{p_0}n_\mu\hat{\Pi}^{L(2n-m)} + \frac{1}{p}\hat{P}_\mu\hat{\Pi}^{C(2n-m)}\right)\hat{\tau}\hat{\Pi}^{(m)\mu}, \end{aligned} \quad (35)$$

$$\begin{aligned} \hat{\Pi}^{L(2n)} &= -\frac{p^2}{p_0^2}\hat{\Pi}^{D(2n)} + 2\frac{p}{p_0}(pn_\mu + p_0\hat{P}_\mu)\hat{\Pi}^{(2n)\mu} + \sum_{m=2}^{2n-2} \left(-\frac{1}{p_0}n_\mu\hat{\Pi}^{L(2n-m)}\right. \\ &\quad \left.+ \frac{p}{p_0^2}\hat{P}_\mu\hat{\Pi}^{D(2n-m)} - \frac{1}{p_0}\left(\frac{p_0}{p}\hat{P}_\mu + n_\mu\right)\hat{\Pi}^{C(2n-m)}\right)\hat{\tau}\hat{\Pi}^{(m)\mu}. \end{aligned} \quad (36)$$

Equations (35) and (36) are also valid for the improved perturbation theory in which the Hard-Thermal-Loops (HTL) resummation is performed for soft modes [11]. Equations (35) and (36) serve as a consistency check of the perturbative computation.

Computation of ghost self-energy part is much simpler than that of gluon self-energy part. This is because the ghost propagator (7) is static and then diagonal in type space. As an illustration, we compute one-loop contribution $\hat{\Pi}_{rs}^{(2)\mu}$. Using Eqs.(7) and (8), we can write $\tilde{\Pi}_{rs}^{(2)ab,\mu}$ as

$$-i\tilde{\Pi}_{rs}^{(2)ab,\mu}(X; P) = gf^{ecb}f^{eac} \int \frac{d^4K}{(2\pi)^4} (P-K)_{\underline{\nu}} \tilde{\Delta}_{rs}(P-K) \Delta_{rs}^{\mu\nu}(K) \quad (37)$$

with no summation over r and s. In Eq.(37), $\tilde{\Delta}_{rs}$ is as in Eq.(7), and $\Delta_{11}^{\mu\nu}(K)$ $[\Delta_{22}^{\mu\nu}(K)]$ is the (11) [(22)]-component of the gluon propagator:

$$\begin{aligned} -i\Delta_{11}^{\mu\nu}(K) &= -i(\Delta_{22}^{\mu\nu}(K))^* \\ &= \mathcal{P}^{T,\mu\nu}(\hat{\mathbf{k}}) \frac{-1}{K^2 + i0^+} + \frac{n^\mu n^\nu}{k^2} - \frac{1}{\lambda} \frac{K^\mu K^\nu}{k^4} \\ &\quad + i\mathcal{P}^{T,\mu\nu}(\hat{\mathbf{k}}) 2\pi\delta(K^2) \left(\theta(k_0)n(X; k, \hat{\mathbf{k}}) + \theta(-k_0)n(X; k, -\hat{\mathbf{k}}) \right). \end{aligned} \quad (38)$$

Here $n(X; k, \hat{\mathbf{k}})$ is the number density of the gluon with momentum \mathbf{k} at the spacetime point X^σ . From Eq.(37) with Eq.(38), one can readily see that $\tilde{\Pi}_{12}^{(2)ab,\mu} = \tilde{\Pi}_{21}^{(2)ab,\mu} = 0$ and $(\tilde{\Pi}_{22}^{(2)ab,\mu})^* = -\tilde{\Pi}_{11}^{(2)ab,\mu}$. Straightforward manipulation yields

$$\hat{\Pi}^{(2)\mu=0} = 0, \quad (39)$$

$$\begin{aligned} \tilde{\Pi}_{11}^{(2)\mu}(X; P) &= \frac{3}{2}g^2 \mathbf{I} \int \frac{d^3k}{(2\pi)^3} \frac{-P^\mu + (\mathbf{p} \cdot \hat{\mathbf{k}})\hat{K}^\mu}{k(\mathbf{p} - \mathbf{k})^2} \left(n(X; k, \hat{\mathbf{k}}) + n(X; k, -\hat{\mathbf{k}}) \right) \\ &+ \dots, \end{aligned} \quad (40)$$

where ‘ \dots ’ stands for the contribution from vacuum theory, which depends on the renormalization scheme. For the isotropic QGP, $n(X; k, \hat{\mathbf{k}}) = n(X; k)$, Eq.(40) turns out to

$$\begin{aligned} \tilde{\Pi}_{11}^{(2)\mu}(X; P) &= -3\pi g^2 \mathbf{I} \hat{P}^\mu \int_0^\infty \frac{dk}{(2\pi)^3} n(X; k) \left[\frac{p^2 + k^2}{pk} + \frac{(p^2 - k^2)^2}{2p^2 k^2} \ln \frac{p - k}{p + k} \right] \\ &+ \dots. \end{aligned} \quad (41)$$

Note that $\tilde{\Pi}_{11}^{(2)\mu}$ in Eqs.(40) and (41) are independent of p_0 . Thus, Eqs.(35) and (36) tell us that for obtaining the components of the gluon self-energy parts, $\hat{\Pi}^{P(2)}(P = L, C, D)$, computation of one of them is sufficient. It is to be noted that the imaginary part of $\hat{\Pi}^L$ on the mass-shell is proportional to the damping rate for longitudinal gluon, which is a propagating mode of gluonic quasiparticle (plasmon) in QGP.

In the case of covariant gauge, $\hat{\Pi}_{\nu\mu}(X; P)$ is usually decomposed as [6]

$$\begin{aligned} \hat{\Pi}_{\nu\mu}(X; P) &= \mathcal{P}_{\nu\mu}^T(\hat{\mathbf{p}})\hat{\Pi}^T(X; P) + \mathcal{P}_{\nu\mu}^L(P)\hat{\Pi}^L(X; P) \\ &+ \mathcal{C}_{\nu\mu}(P)\hat{\Pi}^C(X; P) + \mathcal{D}_{\nu\mu}(P)\hat{\Pi}^D(X; P), \end{aligned} \quad (42)$$

where, $\mathcal{P}_{\nu\mu}^T(\hat{\mathbf{p}})$ is as in Eq.(32) and

$$\mathcal{P}_{\nu\mu}^L(P) \equiv g_{\nu\mu} - \frac{P_\nu P_\mu}{P^2 + i0^+} - \mathcal{P}_{\nu\mu}^T(\hat{\mathbf{p}}), \quad (43)$$

$$\mathcal{C}_{\nu\mu}(P) \equiv \frac{1}{\sqrt{2}p_0 p} \left(P_\nu P_\mu + P_\mu P_\nu + 2p^2 \frac{P_\nu P_\mu}{P^2 + i0^+} \right), \quad (44)$$

$$\mathcal{D}_{\nu\mu}(P) \equiv \frac{P_\nu P_\mu}{P^2 + i0^+}. \quad (45)$$

Note that $\hat{\Pi}^T$, $\hat{\Pi}^L$, $\hat{\Pi}^C$ and $\hat{\Pi}^D$ in Eq.(42) are different from those in Eq.(31).

Substituting Eq.(42) into Eq.(30), we obtain the covariant-gauge counterparts of Eqs.(33) and (34):

$$\hat{\Pi}^C = \sqrt{2}(pn_\mu + p_0 \hat{P}_\mu) \hat{\Pi}^\mu - \left(\frac{\sqrt{2}}{p} \mathcal{P}_{0\mu}^L \hat{\Pi}^L + \frac{P_\mu}{P^2 + i0^+} \hat{\Pi}^C \right) \hat{\tau} \hat{\Pi}^\mu, \quad (46)$$

$$\hat{\Pi}^D = \frac{-1}{P^2 + i0^+} \left(\frac{pn_\mu + p_0 \hat{P}_\mu}{\sqrt{2}} \hat{\Pi}^C + P_\mu \hat{\Pi}^D \right) \hat{\tau} \hat{\Pi}^\mu. \quad (47)$$

Substituting the perturbation series of $\hat{\Pi}_{\nu\mu}$ and $\hat{\Pi}_\mu$ into Eqs.(46) and (47), we obtain

$$\begin{aligned}\hat{\Pi}^{C(2n)} &= \sqrt{2}(pn_\mu + p_0\hat{P}_\mu)\hat{\Pi}^{(2n)\mu} \\ &\quad - \sum_{m=2}^{2n-2} \left(\frac{\sqrt{2}}{p} \mathcal{P}_{0\mu}^L \hat{\Pi}^{L(2n-m)} + \frac{P_\mu}{P^2 + i0^+} \hat{\Pi}^{C(2n-m)} \right) \hat{\tau} \hat{\Pi}^{(m)\mu}, \quad (48) \\ \hat{\Pi}^{D(2n)} &= \frac{-1}{P^2 + i0^+} \sum_{m=2}^{2n-2} \left(\frac{pn_\mu + p_0\hat{P}_\mu}{\sqrt{2}} \hat{\Pi}^{C(2n-m)} + P_\mu \hat{\Pi}^{D(2n-m)} \right) \hat{\tau} \hat{\Pi}^{(m)\mu}.\end{aligned}\quad (49)$$

Some observations are in order.

1. $\hat{\Pi}^{C(2)} = \sqrt{2}(pn_\mu + p_0\hat{P}_\mu)\hat{\Pi}^{(2)\mu}$ and $\hat{\Pi}^{D(2)} = 0$. As in the case of Coulomb gauge, computation of $\hat{\Pi}^{(2)\mu}$ is relatively easy.
2. $\hat{\Pi}^{C(2n)}$ is written in terms of $\hat{\Pi}^{(2m)\mu}$ and $\hat{\Pi}^{L(2m-2)}$ with $m \leq n$.
3. $\hat{\Pi}^{D(2n)}$ starts from $\hat{\Pi}^{D(4)}$ and is written in terms of $\hat{\Pi}^{(2m)\mu}$ and $\hat{\Pi}^{L(2m-2)}$ with $m \leq n-1$.

5 Acknowledgements

I would like to thank Prof. A. Niégawa for helpful discussions and careful reading of the manuscript.

References

- [1] See. e.g., T. Hatsuda, Y. Miake, S. Nagamiya, K. Yagi (eds.), *Nucl. Phys.* **A638**, 1 (1998); *Proc. 48th Yamada Conference, Quark Matter '97* (Tsukuba, December 1-5, 1997); U. Heinz (ed.), /hep-ph/9811469/tft98proceedings.html, *TFT98: Proc. 5th International Workshop on Thermal Field Theories and their Applications* (Regensburg, August 10-14, 1998).
- [2] M. Dirks, A. Niégawa, K. Okano, *Phys. Lett.* **B461**, 131 (1999).
- [3] J. I. Kapusta, *Finite-Temperature Field Theory* (Cambridge Univ. Press, 1989).
- [4] M. Le Bellac, *Thermal Field Theory* (Cambridge Univ. Press, Cambridge, 1996).
- [5] J. Schwinger, *J. Math. Phys.* **2**, 407 (1961); P. M. Bakshi and K.T. Mahanthappa, *ibid.* **4**, 1, 12 (1963); L. V. Keldysh, *Zh. Eksp. Teor. Fiz.* **47**, 1515 (1964) [*Sov. Phys. -JETP* **20**, 1018 (1965)]; R. A. Craig, *J. Math. Phys.* **9**, 605 (1968); J. Rammer and H. Smith, *Rev. Mod. Phys.* **58**, 323 (1986); E. Calzetta and B. L. Hu, *Phys. Rev.* **D37**, 2878 (1988); M. Morikawa, *Prog. Theor. Phys.* **93**, 685 (1995).
- [6] N. P. Landsman and Ch. G. van Weert, *Phys. Rep.* **145**, 141 (1987).
- [7] K.-C. Chou, Z.-B. Su, B.-L. Hao and L. Yu, *Phys. Rep.* **118**, 1 (1985).

- [8] H. Umezawa, *Advanced Field Theory-Micro, Macro, and Thermal Physics* (AIP, New York, 1993); H. Umezawa, H. Matsumoto and M. Tachiki, *Thermo Field Dynamics and Condensed States* (North-Holland, Amsterdam, 1982); Y. Yamanaka, H. Umezawa, K. Nakamura and T. Arimitsu, *Int. J. Mod. Phys.* **A9**, 1153 (1994); Y. Yamanaka and K. Nakamura, *Mod. Phys. Lett.* **A9**, 2879 (1994); H. Chu and H. Umezawa, *Int. J. Mod. Phys.* **A9**, 1703, 2363 (1994).
- [9] H. Hata, T. Kugo, *Phys. Rev.* **D21**, 3333 (1980); H. Matsumoto, Y. Nakano, H. Umezawa, *ibid.* **D28**, 1931 (1983).
- [10] C. Itzykson, J.-B. Zuber, *Quantum Field Theory*, (McGraw-Hill, New York, 1980).
- [11] R.D. Pisarski, *Phys. Rev. Lett.* **63**, 1129 (1989); E. Braaten, R.D. Pisarski, *Nucl. Phys.* **B337**, 569 (1990); **B339**, 310 (1990); J. Frenkel, J.C. Taylor, *ibid.* **B334**, 199 (1990).